A Hard Constraint on the Efficacy of Serological Tests

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2020-04-05

Consider an infectious disease D with a basic reproduction number of R_0. Suppose there is a serological test T such that people who test T+ are considered immune to D, and those who test T– are considered susceptible to D. The test T thus defines two subpopulations, T+ and T–. However, there are thereby four nominal subpopulations implicitly defined: true T+, false T+, true T– and false T–.

(For subpopulations U and V, I use below the notation U+V to denote the subpopulation consisting of those in either U or V, that is, the union of the subpopulations.)

Suppose a society in the grip of D has established control measures, namely that people in T– self-isolate, but people in T+ may commune freely. Let us assume these control measures are exceptionlessly efficacious, namely that R_t in T– is 0.

The implicitly defined subpopulations are:

- True-positive T+. Call these I (for "immune")
- False-positive T+. Call these Sf (for "susceptible false-immune")
- True-negative T–. These are susceptible S.
- False-negative T–. These are in fact immune, thus I.

There are thus three factual subpopulations of interest: I, Sf, S. Of these, I are immune to D, and Sf+S are susceptible to D.

Since self-isolation is in force, R_t for S+I is 0.

Sf+I may mingle freely. The Sf proportion is still susceptible to D, whereas the I proportion not.

Suppose T is efficacious in proportion x. That is, T yields T+ in I in proportion x and T+ in Sf in proportion (1 – x). These populations, having tested T+, mingle freely.

Suppose one infected person, called Inf, in T+ mingles freely with those in Sf+I. Inf can infect only those in Sf.

Suppose the time unit is chosen to be identical with the serial interval. Then in one time unit, Inf infects R_0 people in Sf, that is, $(1 – x).R_0$ people.

It is well-understood that, in order to dampen D, this number $(1 – x).R_0$ must be < 1.

This constraint yields $(1 – x) < 1/R_0$ and thus $x > 1 – 1/R_0$.

Conclusion: in order to dampen D with basic reproductive number R_0, assuming perfect sociological compliance with self-isolation requirements, a serological test must yield proportionally at least $(1 – 1/R_0)$ true positive results.

Let us take as an illustrative example Covid-19.

For D with R_0 of 2.3, which is a lower bound for Covid-19, $x > 0.565$. That is, at least 56.5% of T positives must be true positives. For a D with a higher R_0, which is (at time of writing) plausible for Covid-19, the accuracy of serological test T must be correspondingly higher.

This simple arithmetic must surely be well-known, but so far it has not appeared in the literature.